Long-time signatures of short-time dynamics in decaying quantum-chaotic systems

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We analyze the decay of classically chaotic quantum systems in the presence of fast ballistic escape routes on the Ehrenfest time scale. For a continuous excitation process, the form factor of the decay cross section deviates from the universal random-matrix result on the Heisenberg time scale, i.e., for times much larger than the time for ballistic escape. We derive an exact analytical description and compare our results with numerical simulations for a dynamical model.

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Properties of complex quantum systems are mostly characterized by a high degree of universality which is rooted in the random interference of many partial waves [1]. A prototypical example is the cross section fluctuations which arise from the random interference between the decay modes of an excited quantum system. In the case of a large number of decay channels, these fluctuations are free of all characteristics except for the classical decay rate (Ericson fluctuations) [2–4]. The main theoretical tool to describe such universal properties is random-matrix theory (RMT) [5,6]. Semiclassical theories reveal limits to this description which are rooted in direct processes and hence best characterized in the time domain. Most notably, if the complex quantum dynamics is due to a chaotic classical limit, the random wave interference is only established after a finite time, the so-called Ehrenfest time [7]. For shorter times, the quantum dynamics follows the classical dynamics quasideterministically. Over recent years, a large number of studies have explored the consequences of a finite Ehrenfest time [8–11], but almost exclusively these studies were concerned with stationary properties.

The purpose of this paper is to develop a dynamical theory for a physical observable—the form factor of the decay cross section of an excited quantum system—in which nonuniversal corrections can be detected directly in the time domain. The dynamical theory is based on a stroboscopic description which extends the previous analysis of stationary properties in systems with a finite Ehrenfest time [12,13]. In the absence of direct decay processes, our model exhibits the typical universal fluctuations found in autonomous models of quantum decay and described by RMT [14,15]. We incorporate quasideterministic direct decay by applying a mapping formalism which has earlier been used to describe Fano resonances [16] due to direct decay processes [17–19]. It turns out that the deviations in the form factor from the universal behavior persist up to times comparable to the Heisenberg time, hence far beyond the regime of quasideterministic decay.

Molecular photodissociation and atomic autoionization are examples of half-collision processes (see Ref. [20] and references therein), i.e., the coherent dynamics of quantum systems which have been excited by external means (e.g., the absorption of a photon) and decay subsequently via energetically accessible open channels. Within the half-collision description, each decay amplitude is given by the overlap $\langle\Psi_a|\alpha\rangle$ of the square integrable initial state $|\alpha\rangle$ and the scattering state $|\Psi_a\rangle$ associated with channel a. The form factor $\hat{C}(t)$ is defined as the Fourier transform of the autocorrelation function of the total decay cross section $\sigma(E)=\Sigma_a|\langle\Psi_a|\alpha\rangle|^2$. It can also be obtained from the Fourier transform of the cross section itself,

$$\hat{C}(t)=\frac{1}{L}|\hat{\sigma}(t)|^2,\quad \hat{\sigma}(t)=\int_{-L/2}^{L/2}dE\,e^{-2\pi iEt}\sigma(E). \tag{1}$$

Here, we measure time in units of the Heisenberg time $t_H=h/\Delta$, and energy in units of the mean level spacing Δ (h is Planck's constant). Typically, only a finite energy range, classically small but quantum mechanically large ($L\gg 1$), enters this formula.

In previous works [14,15,17–19], the random-matrix description of half-collision processes has been set up for autonomous systems which are described by an effective Hamiltonian [1]. In the case of time-reversal invariance, the autocorrelation function then follows from the Verbaarschot-Weidenmüller-Zirnbauer (VWZ) integral [21], and for systems without direct decay the form factor is given by

$$C_0(t)=\int_{\max(0,t-1)}^{t}dr\int_0^r\frac{du}{2u+1}\frac{4(t-r)(r+1-t)^{1+N}f_0}{(t^2-r^2+x)^2(1+2r+x)^{N/2}},$$

$$x=u^2\frac{2r+1}{2u+1},\quad f_0=r^2+2rt+t-x. \tag{2}$$

Stroboscopic model. We base our dynamical theory of half-collision processes on a time-periodic quantum map description which has been previously used to describe transport in systems with a finite Ehrenfest time t_{Ehr} [12,13,22–24]. Because of time-reversal invariance of the decay dynamics, the Floquet operator F is symmetric, $F=F^T$, which allows us to decompose F into $F=F_{\mathrm{out}}F_{\mathrm{in}}$, with $F_{\mathrm{in}}{}^T=F_{\mathrm{out}}$ [5]. To introduce decay, we specify an N-dimensional subspace (spanned by the column vectors of the matrix p) within the M-dimensional Hilbert space of F. That subspace provides the interface between the closed system and the

decay channels. For ballistic decay (ideal coupling) we define $Q=1-pp^T$ and obtain the time-reversal symmetric open quantum map [12,13]

$$\psi(n+1) = F_{\text{in}} Q F_{\text{out}} \psi(n), \tag{3}$$

which describes how the internal wave function ψ is degraded sequentially in successive decay attempts (where the integer n is the stroboscopic time).

If we consider a short excitation pulse which leaves the system in the internal initial state $|\alpha\rangle=\psi(0)$, the quantum map (3) yields

$$\psi(n) = \begin{cases} (F_{\text{out}}^{\dagger} Q F_{\text{in}}^{\dagger})^{-n}|\alpha\rangle, & n \leqslant 0, \\ (F_{\text{in}} Q F_{\text{out}})^{n}|\alpha\rangle, & n \geqslant 0, \end{cases} \tag{4}$$

for the forward and reversed time evolution of the system. By contrast, for a continuous excitation process, Eq. (3) implies [25]

$$\sigma(\varepsilon) = 1 + 2\,\text{Re}\langle\alpha|F_{\text{in}} Q F_{\text{out}} \frac{1}{e^{-i\varepsilon} - F_{\text{in}} Q F_{\text{out}}}|\alpha\rangle. \tag{5}$$

Both types of experiments are related to one another by the fact that the Fourier transform of $\sigma(\varepsilon)$ is just the return amplitude. That is, in the energy and time units of Eq. (1), $\hat{\sigma}(t)=M\langle\alpha|\psi(n)\rangle$, where $t=n/M$. The stroboscopic form factor is then obtained from Eq. (1). Note that the spectrum of F is homogeneous on the unit circle, so that we may set $L=M$.

Direct processes. We now incorporate quasideterministic direct processes by assuming that part of the initial amplitude escapes during the first iteration of the map. Note that the total cross section contains the forward but also the backward (time-reversed) evolution Eq. (4). This means that the autocorrelation function will be equally sensitive to direct decay in either direction. To study this dependence quantitatively, we assume the following decomposition for the initial state:

$$|\alpha\rangle = |\alpha_0\rangle + F_{\text{out}}^{\dagger} p|\alpha^+\rangle + F_{\text{in}} p|\alpha^-\rangle. \tag{6}$$

Here, $|\alpha_0\rangle$ and $|\alpha^{\pm}\rangle$ are assumed to be real in the basis chosen. Also, we assume that the three terms are approximately orthogonal to each other, e.g., $\langle\alpha^+|p^{\dagger}Fp|\alpha^-\rangle\approx 0$, which would follow from the absence of any prompt processes in the full scattering system.[1] Hence, we find $\|\alpha\|^2=\|\alpha_0\|^2+\|\alpha^-\|^2+\|\alpha^+\|^2$. While $|\alpha_0\rangle$ leads to purely indirect decay, $\|\alpha^{\pm}\|^2$ gives the probability for direct decay within the first step of the open map, Eq. (3), forward (backward) in time. If $|\alpha^+\rangle=|\alpha^-\rangle$, $|\alpha\rangle$ becomes real, and the whole decay process becomes symmetric in time. The orthogonality of the three terms in Eq. (6) then implies that the maximum amount of direct decay (in forward or backward evolution) is restricted to one-half.

To make contact with RMT we assume that F_{out} is taken from the circular unitary ensemble, which then implies that

[1]In order that $\langle\alpha^+|p^{\dagger}Fp|\alpha^-\rangle$ is different from zero, F must map parts of the p subspace onto itself. That implies a prompt component in the full scattering process.

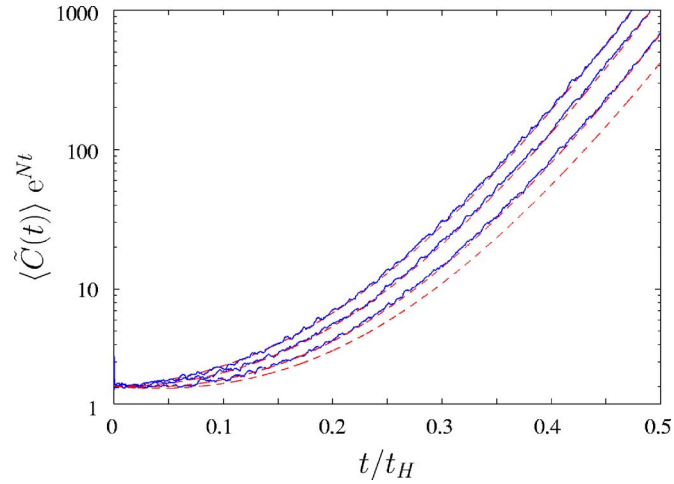

FIG. 1. (Color online) Form factor $\langle\tilde{C}(t)\rangle$ times e^{Nt} for the case of time-reversal-invariant decay (real initial coherent wave packet in the open kicked rotor) and for different amounts of overlap of the initial state with the regions of fast decay (solid lines). Here, $M=4096$ and $N=50$. The corresponding theoretical results [Eq. (7), with $|\alpha^+\rangle=|\alpha^-\rangle$] are shown with dashed lines. From top to bottom $\|\alpha^{\pm}\|^2=0,0.25,0.417,0.5$.

$F=F_{\text{out}}F_{\text{out}}^T$ is a member of the circular orthogonal ensemble [5]. The resulting scattering ensemble [24] is essentially equivalent to the random Hamiltonian ensemble discussed in [1]. In particular, the S-matrix correlations are well described by the VWZ integral [21,26]. This allows us to calculate the autocorrelation function of the cross section in the mapping formalism of Refs. [18,19]. The result is (for details see [27])

$$\hat{C}(t) = \|\alpha_0\|^4 C_0(t) + \frac{1}{2}\|\alpha_0\|^2(\|\alpha^+\|^2 + \|\alpha^-\|^2 + 2\langle\alpha^+|\alpha^-\rangle)C_{01}(t) + \frac{1}{4}(\|\alpha^+\|^2\|\alpha^-\|^2 + \langle\alpha^+|\alpha^-\rangle^2)C_{11}(t), \tag{7}$$

where $C_{01}(t)$ and $C_{11}(t)$ are given by the expression (2), but with the function f_0 replaced by $f_{01}=t$ and $f_{11}=(x+r)/(1+2r+x)+(t-r)/(1+r-t)$, respectively. The auxiliary correlation functions $C_0(t)$, $C_{01}(t)$, and $C_{11}(t)$ all tend to 2 as $t\to 0$. The functions $C_0(t)$ and $C_{11}(t)$ can be seen in Fig. 1. They correspond to the limit cases of purely indirect ($\|\alpha^{\pm}\|^2=0$) and maximally direct ($\|\alpha^{\pm}\|^2=0.5$) decay. The function $C_{01}(t)$, if plotted, would lie in between. On a time scale given by $t_H \gg t_{\text{Ehr}}$, all three functions are notably different. Equation (7) is the central result of this paper. It generalizes related results obtained within the autonomous random-matrix model [18,19]. These results are recovered (for ballistic decay) by choosing $|\alpha^+\rangle=|\alpha^-\rangle$, while the universal random-matrix prediction (2) is obtained for $|\alpha^{\pm}\rangle=0$. In general, the form factor depends on three independent parameters, the probabilities $\|\alpha^{\pm}\|^2$ for direct decay in forward (backward) time evolution as well as the angle between the two corresponding amplitude vectors.

Maximally asymmetric decay is obtained by setting $|\alpha^-\rangle$ to zero. It yields

$$\hat{C}(t) = \|\alpha_0\|^4 C_0(t) + \frac{1}{2}\|\alpha_0\|^2\|\alpha^+\|^2 C_{01}(t). \tag{8}$$

This expression also describes the decay form factor for an initial state $|\alpha\rangle = F_{\text{in}}(|\alpha_0\rangle + p|\alpha^-\rangle)$, and yields the same dynamics as choosing the initial state $|\alpha_0\rangle + p|\alpha^-\rangle$ for the open map $\chi(n+1) = QF\chi(n)$ used, e.g., in Refs. [13,28].

Numerical results. The decay form factor of realistic physical systems can be obtained from experiments or in sophisticated numerical computations [29]. Here we test our theoretical predictions by numerical investigations of a simple model system, the open kicked rotator [12,13], which was used in most studies of the stationary properties of quantum systems with a finite Ehrenfest time [8,10,9]. In the position basis the Floquet matrix can be decomposed as $F = XU^\dagger \Pi U X$, where

$$\Pi = \text{diag}(e^{-i\pi k^2/M}),$$

$$X = \text{diag}\{\exp[-iMV(2\pi k/M)]\}, \tag{9}$$

and $V(\theta) = K(\cos\theta - \gamma \sin 2\theta)$ with $\gamma = 1$. For $\gamma = 0$, the standard kicked rotator, there are three different symmetries present: time reversal, reflection, and conjugation (the combination of time reversal and reflection). A finite value of γ breaks the reflection and also the conjugation symmetry. The initial wave packet $|\alpha\rangle$ is constructed as a superposition of two coherent states, located around the line $p = 0$ and therefore real. The amount of overlap with the regions of fast decay (equal for both backward and forward evolution) was controlled by changing the location of and/or squeezing the coherent states.

We analyze the normalized autocorrelation function $\tilde{C}(t) = 2\hat{C}[\sigma](t)/\hat{C}[\sigma](0)$, whose limit value at small times is 2, independent of the probabilities for direct decay. This makes sure that the asserted sensitivity to direct processes is related to the shape of the autocorrelation function, not its norm (in practice, the cross sections measured or calculated usually lack a reliable absolute scale).

We first consider the case of a real-valued initial state, with symmetric decay in time. Figure 1 shows the results for the form factor $\langle \tilde{C}(t) \rangle$, scaled by the classical survival probability e^{-Nt}. The numerical results are obtained for a large number of internal modes $M = 4096$ (required for our random-matrix model to apply) and a large number of open channels $N = 50$. This allowed us to reach $\|\alpha^\pm\|^2 = 0.417$ as the maximal probability for direct decay. We take averages over 100 different realizations of the system, each corresponding to a different value of the kick strength K (for all of them the classical dynamics is completely chaotic). The theoretical results derived from Eq. (7) with parameters corresponding to the numerical data are shown with dashed lines. The numerical results agree perfectly well with the theoretical predictions, within the remaining statistical uncertainty.

So far we have only discussed an ensemble average, while experimentally or numerically it is not always possible or desirable to change any system parameters. Can one detect the direct quasi-deterministic decay for an individual system? This question is addressed, for the case of maximally asymmetric decay, in Fig. 2. We show the results for $M = 10\,000$ and two values of N: $N = 20$ in Figs. 2(a)–2(c) and 50 in Figs. 2(d)–2(f). On the vertical axis we plot the function $Y(t)$ which we obtain from the numerical data through the transformation [27]

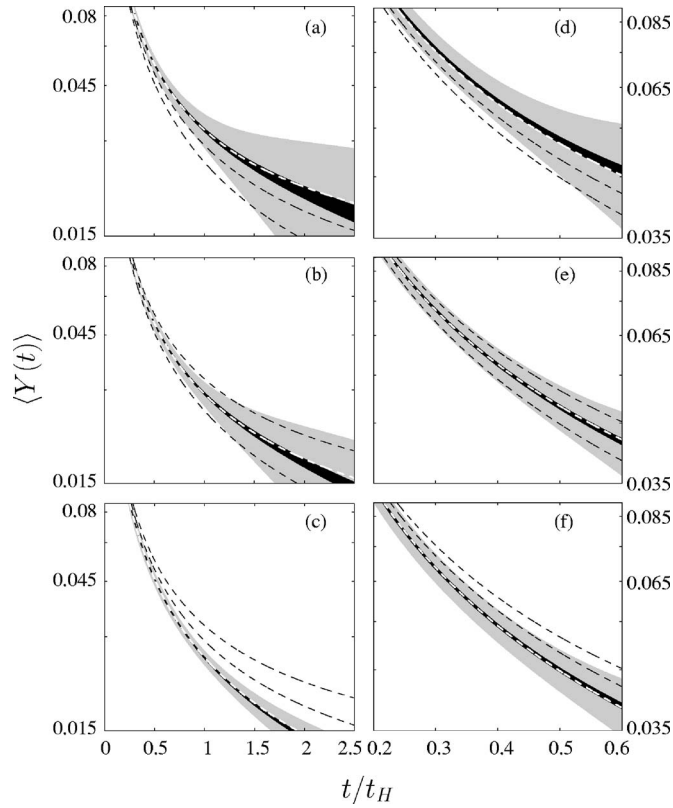

FIG. 2. Numerical data and theory for $Y(t)$, for $N = 20$ channels (left panels) and $N = 50$ channels (right panels). The overlap of the initial state with the regions of fast decay is given by $\|\alpha^-\|^2 = 0$ (a), 0.595 (b), 0.907 (c), 0 (d), 0.571 (e), and 0.906 (f). Areas in gray represent the range of the standard deviation of $Y(t)$, while areas in black represent the standard deviation for $\langle Y(t) \rangle$ when the average is taken over 100 samples. The dashed lines show the theoretical curves for the different values for $\|\alpha^-\|^2$; the respective theoretical curve corresponding to the numerical result is plotted in light gray.

$$Y(t) = \frac{1}{t}\sum_{m=20}^{Nt} \frac{\tilde{C}(m/M)}{[C_0(m/M) - C_{01}(m/M)]}, \tag{10}$$

which allows one to obtain a best fit value for $\|\alpha^-\|^2$ by linear regression (not shown). In this sum we have discarded the first 20 kicks to exclude system-specific processes at the Ehrenfest time scale. It removes the fluctuations in time, so that only the sample-to-sample fluctuations survive. These are shown by the gray areas in Fig. 2, while the expected standard deviation for the average over 100 samples is shown in black. The dashed lines show the different theoretical curves, obtained from Eq. (8), that fit best each one of the numerical results. Since the sample-to-sample fluctuations (light gray) often do not cover the whole available range between the theory for purely indirect and dominantly direct decay, even a single experiment may be sufficient to

extract some information on the presence of direct decay processes.

We have developed a dynamical description for the form factor $\hat{C}(t)$ of a decaying quantum system, taking into account the effect of direct (sub-Ehrenfest-time) decay processes. We derived a general analytical expression for $\hat{C}(t)$ at times of the order of the Heisenberg time, on the basis of a suitable random-matrix model. While earlier studies [14,15,17–19] implicitly assumed that the wave packet will follow the same dynamics in forward and backward evolution, we allow for excitation mechanisms which lead to asymmetric decay in time. In physical terms, asymmetric decay would result from the preparation of an initial wave packet with finite group velocity. Properly directed this might strongly enhance the probability for prompt decay, but only in one direction in time.

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